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Natural convection with variable viscosity and thermal conductivity from a vertical wavy cone

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Abstract — The effect of variable viscosity and thermal conductivity on natural convection over an isothermal vertical wavy cone is studied in this paper. We consider the boundary-layer regime having larger Grashof number and assume the wavy surfaces with O(1) amplitude and wavelength. Using the appropriate variables the basic equations are transformed to nonsimilar boundary-layer equations which reduce the wavy cone to a flat one. These equations are then solved numerically using a very efficient implicit finite-difference method known as Keller box scheme. Detailed results for the streamlines, isotherms, reduced skin friction and heat transfer rates for a selection of parameter sets consisting of the viscosity parameter, thermal conductivity parameter, wavy surface amplitude and half cone angle. © 2001 Éditions scientifiques et médicales Elsevier SAS

natural convection / variable viscosity / variable thermal conductivity / Keller box / wavy cone surface

Nomenclature

a	amplitude of the wavy surface of the cone	
c_f	skin friction coefficient	
c_p	specific heat at constant pressure	$J{\cdot}kg^{-1}{\cdot}K^{-1}$
g	acceleration due to the gravity	$m{\cdot}s^{-1}$
Gr	Grashof number	
L	characteristic length associated with the wavy surface	m
Nu	local Nusselt number	
P	pressure	
Pr	Prandtl number	
r	local radius of the flat surface of the cone	
T	temperature	
u, v	velocity components along x, y axes	
x, y	Cartesian coordinates along and normal to the flat surface of the cone, respectively	

Greek symbols

β	coefficient of thermal expansion	K^{-1}
ε	viscosity variation parameter	
γ	thermal conductivity variation parameter	
$\mu(T)$	variable viscosity	$kg{\cdot}m^{-1}{\cdot}s^{-1}$
$\kappa(T)$	variable thermal conductivity	$W{\cdot}m^{-1}{\cdot}K^{-1}$
$\sigma(x)$	surface geometry function	
θ	reduced temperature	
η	pseudo-similarity variables	
ρ	density	$kg{\cdot}m^{-3}$
ϕ	cone half-angle	
ψ	stream function	$m^2{\cdot}s^{-1}$

Subscripts

w	wall conditions
∞	ambient conditions
x	differentiation with respect to x

Superscripts

–	dimensional variables
′	differentiation with respect to η

1. INTRODUCTION

The problem of laminar natural convection boundary layer flow and heat transfer over a vertical full cone get a great deal of attention in various branches of engineering. If the surface is roughened the flow is disturbed by the surface and this alters the rate of heat trans-

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fer. These types of roughened surface are taken into account in several heat transfer collectors and flat plate condensers in refrigerators. In cavity wall insulting systems and grain storage containers the surfaces are nonuniform in large scale. Yao [1], and Moulic and Yao [2] have studied such nonuniformities on the vertical convective boundary layer flow of a Newtonian fluid. Hossain and Pop [3] investigated the magnetohydrodynamic boundary layer flow and heat transfer from a continuous moving wavy surface, while the problem of free convection flow from a wavy vertical surface in the presence of a transverse magnetic field was studied by Hossain et al. [4]. On the other hand, the free convection boundary layer induced by vertical and horizontal surfaces exhibiting small-amplitude waves and which are embedded in a porous medium were investigated by Rees and Pop [5–7]. Recently, Hossain and Rees [8] have considered the combined effects of thermal and mass diffusion on the natural convection flow of a viscous incompressible fluid from a vertical wavy surface. In this paper they studied the effects of waviness of the surface on the heat flux and mass flux distributions in combination with the species concentration for a fluid having Prandtl number 0.7. Very recently, Pop and Na [9] studied both the constant wall temperature and constant heat flux distribution on the natural convection flow over a vertical wavy frustum of a cone.

All the above authors [1–9] considered the fluids having uniform viscosity and thermal conductivity throughout the flow regime. But, these physical properties may be changed significantly with temperature. Gary et al. [10] and Mehta and Sood [11] have found that the flow characteristics substantially change when the effect of viscosity is included. After these, the effect of temperature dependent viscosity on the mixed convection flow from a vertical flat plate in the region near the leading edge was investigated by Hady et al. [12], Kafoussius and Williams [13] and Kafoussias and Rees [14]. Very recently, Hossain et al. [15–17] have investigated the natural convection flow from a vertical wavy surface, a truncated cone and a wavy cone, respectively. In all the above studies [13–17] the viscosity of the fluid has been considered to be inversely proportional to a linear function of temperature. Besides these, it has been found that for liquid, such as air, the viscosity μ varies with temperature in an approximately linear manner. For a liquid, it also has been found that the thermal conductivity κ varies with temperature in an approximately linear manner in the range from 0 to 400 °F (see Kays [18]). A semi-empirical formula for the thermal conductivity was used by Arunachalam et al. [19]. Following these authors, Chaim [20] has investigated the effect of a variable thermal conductivity over a linearly stretching sheet. Assuming the viscosity and thermal conductivity of the fluid to be proportional to a linear function of temperature, two semi-empirical formulae were proposed by Charraudeau [21]. Following him, Hossain et al. [22, 23] investigated the mixed convection along a vertical flat plate, the natural convection past a truncated cone and for a wedge flow for the fluid having temperature dependent viscosity and thermal conductivity.

Here we investigate the free convection boundary layer over a vertical wavy cone maintained at a uniform surface temperature immersed in a fluid with a temperature dependent viscosity and thermal conductivity. The transformed boundary-layer equations are solved by using finite-difference method (Keller [24]). We give our attention to the situation where the buoyancy forces assist the flow for various values of the viscosity variation parameter ε and the thermal conductivity variation parameter γ with the Prandtl number $Pr = 0.7$, which is appropriate for air. From these results we can observe the different flow and heat transfer characteristics by varying the relevant parameters.

2. FORMULATION OF THE PROBLEM

A steady two-dimensional laminar free convective flow of a viscous and incompressible fluid having temperature dependent viscosity and thermal conductivity is considered. Under the usual Boussinesq approximation, the equations governing the flow can be written as

$$\frac{\partial(\hat{r}\hat{u})}{\partial\hat{x}} + \frac{\partial(\hat{r}\hat{v})}{\partial\hat{y}} = 0 \tag{1}$$

$$\hat{u}\frac{\partial\hat{u}}{\partial\hat{x}} + \hat{v}\frac{\partial\hat{u}}{\partial\hat{y}} = -\frac{1}{\rho}\frac{\partial\hat{p}}{\partial\hat{x}} + \frac{1}{\rho}\nabla(\mu\nabla\hat{u}) + g\beta(T - T_\infty)\cos\phi \tag{2}$$

$$\hat{u}\frac{\partial\hat{v}}{\partial\hat{x}} + \hat{v}\frac{\partial\hat{v}}{\partial\hat{y}} = -\frac{1}{\rho}\frac{\partial\hat{p}}{\partial\hat{y}} + \frac{1}{\rho}\nabla(\mu\nabla\hat{v}) - g\beta(T - T_\infty)\sin\phi \tag{3}$$

$$\hat{u}\frac{\partial T}{\partial\hat{x}} + \hat{v}\frac{\partial T}{\partial\hat{y}} = \frac{1}{\rho c_p}\nabla(\kappa\nabla T) \tag{4}$$

where $(\hat{u}, \hat{v})$ are the velocity components along the $(\hat{x}, \hat{y})$ axes, ∇^2 is Laplacian operrator, g is the acceleration due to gravity, ρ is the density, $\mu(T)$ is the viscosity, $\kappa(T)$ is the thermal conductivity of the fluid depending on the fluid temperature T and the temperature of the

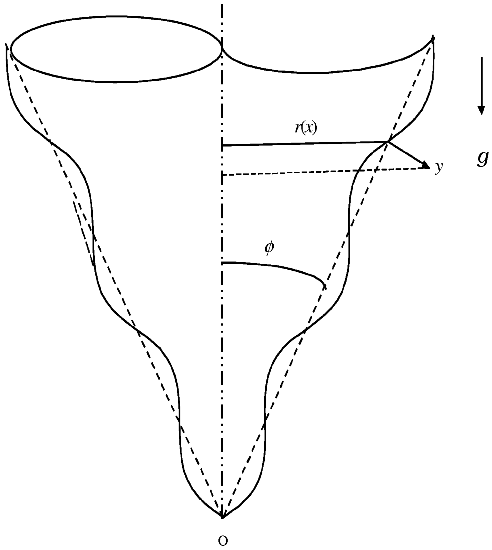

Figure 1. The geometry and the coordinate system.

cone surface is held constant at T_w which is higher than the ambient temperature T_∞. Also ϕ and $\hat{r}$ are the half-angle and the local radius of the flat surface of the cone, where $\hat{r}$ is given by $\hat{r} = \hat{x}\sin\phi$. The boundary layer analysis outlined below allows $\hat{\sigma}(\hat{x})$ to be arbitrary, but our detailed numerical work will assume that the wavy surface of the cone is described by the equation $\hat{y}_w = \hat{\sigma}(\hat{x}) = \hat{a}\sin(2\pi\hat{x})$.

The boundary conditions appropriate for equations (1)–(4) are

$$\begin{aligned} &\hat{u} = 0, \quad \hat{v} = 0, \qquad T = T_w \quad \text{at } \hat{y} = \hat{y}_w = \hat{\sigma}(\hat{x}) \\ &\hat{u} = 0, \quad T = T_\infty, \quad p = p_\infty \quad \text{at } \hat{y} \to \infty \end{aligned} \tag{5}$$

There are very few forms of viscosity and thermal conductivity variations available in the literature. Among them we have considered that one which is appropriate for liquid, introduced by Charraudeau [21] as follows:

$$\begin{aligned} \mu &= \mu_\infty\left[\frac{1+\varepsilon(T-T_\infty)}{T_w - T_\infty}\right] \\ \kappa &= \kappa_\infty\left[\frac{1+\gamma(T-T_\infty)}{T_w - T_\infty}\right] \end{aligned} \tag{6}$$

where μ_∞ and κ_∞ are the viscosity and the thermal conductivity of the ambient fluid and ε and γ are the viscosity variation parameter and thermal conductivity variation parameter. Now we introduce the following nondimensional boundary layer variables:

$$\begin{aligned} &x = \frac{\hat{x}}{L}, \quad y = \frac{\hat{y}-\hat{\sigma}}{L}Gr^{1/4} \\ &r = \frac{\hat{r}}{L}, \quad p = \frac{L^2}{\rho\nu_\infty^2}Gr^{-1}\hat{p} \\ &u = \frac{\rho L}{\mu_\infty}Gr^{-1/2}\hat{u}, \quad v = \frac{\rho L}{\mu_\infty}Gr^{-1/4}(\hat{v} - \sigma_x\hat{u}) \\ &\theta = \frac{T-T_\infty}{T_w - T_\infty}, \qquad Gr = \frac{g\beta(T_w - T_\infty)\cos\phi^3}{\nu_\infty^2}L^3 \\ &a = \frac{\hat{a}}{L}, \quad \sigma(x) = \frac{\hat{\sigma}(x)}{L}, \quad \sigma_x = \frac{d\hat{\sigma}}{d\hat{x}} = \frac{d\sigma}{dx} \end{aligned} \tag{7}$$

where L is the characteristic length associated with the wavy surface of the cone, and ν_∞ $(= \mu_\infty/\rho)$ is the reference kinematic viscosity and Gr is the Grashof number. Introducing these transformations into equations (1)–(5) and ignoring terms of small orders in Gr, we obtain the following boundary layer equations:

$$\frac{\partial(ru)}{\partial x} + \frac{\partial(rv)}{\partial y} = 0 \tag{8}$$

$$\begin{aligned} u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} &= -\frac{\partial p}{\partial x} + Gr^{1/4}\sigma_x\frac{\partial p}{\partial y} + \frac{(1+\sigma_x^2)}{1+\varepsilon\theta}\frac{\partial^2 u}{\partial y^2} \\ &\quad - \frac{\varepsilon(1+\sigma_x^2)}{(1+\varepsilon\theta)^2}\frac{\partial\theta}{\partial y}\frac{\partial u}{\partial y} + \theta \end{aligned} \tag{9}$$

$$\begin{aligned} \sigma_x\left(u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y}\right) - \sigma_{xx}u^2 &= -Gr^{1/4}\frac{\partial p}{\partial y} + \sigma_x\left(1+\sigma_x^2\right)(1+\varepsilon\theta)\frac{\partial^2 u}{\partial y^2} \\ &\quad + \varepsilon\sigma_x\left(1+\sigma_x^2\right)\frac{\partial\theta}{\partial y}\frac{\partial u}{\partial y} - \theta\tan\phi \end{aligned} \tag{10}$$

$$\begin{aligned} u\frac{\partial\theta}{\partial x} + v\frac{\partial\theta}{\partial y} &= \frac{1}{Pr}\left(1+\sigma_x^2\right)(1+\gamma\theta)\frac{\partial^2\theta}{\partial y^2} \\ &\quad + \frac{1}{Pr}\left(1+\sigma_x^2\right)\gamma\left(\frac{\partial\theta}{\partial y}\right) \end{aligned} \tag{11}$$

The corresponding boundary conditions (5) become

$$\begin{aligned} &u = 0, \quad v = 0, \qquad \theta = T_w \quad \text{at } y = 0 \\ &u = 0, \quad T = T_\infty, \quad p = 0 \quad\; \text{as } y \to \infty \end{aligned} \tag{12}$$

Equations (9)–(12) expressed the induced convection for the wavy surface. From equation (10) it can be found that the pressure gradient along the y direction is $O(Gr^{-1/4})$, which indicates that the lowest order of pressure gradient along x direction can be determined from the solution of the inviscid flow. For present study, since there is no externally induced free stream, so this pressure gradient is zero. Equation (10) also shows that $Gr^{1/4}\partial p/\partial y$ is O(1) and is determined by the left-hand side of this equation. Thus, if we eliminate $\partial p/\partial y$ from equations (9) and (10) we get

$$\begin{aligned} u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y} &= \left(1+\sigma_x^2\right)(1+\varepsilon\theta)\frac{\partial^2 u}{\partial y^2}-\frac{\sigma_x\sigma_{xx}}{1+\sigma_x^2}u^2 \\ &\quad +\varepsilon\left(1+\sigma_x^2\right)\frac{\partial u}{\partial y}\frac{\partial \theta}{\partial y}+\left(\frac{1-\sigma_x\tan\phi}{1+\sigma_x^2}\right)\theta \end{aligned} \tag{13}$$

Now we introduce the further transformations into equations (9), (11) and (13) of the form:

$$\begin{aligned} \psi &= x^{3/4} r f(x,\eta), \quad \eta = x^{-1/4} y \\ \theta &= \theta(x,\eta), \quad r = x\sin\phi \end{aligned} \tag{14}$$

where ψ is the stream function which is defined according to $u=(1/r)(\partial\psi/\partial y)$ and $v=-(1/r)(\partial\psi/\partial x)$. The boundary layer equations now become

$$\begin{aligned} &\left(1+\sigma_x^2\right)(1+\varepsilon\theta)f'''+\frac{7}{4}ff''-\left(\frac{1}{2}+\frac{x\sigma_x\sigma_{xx}}{1+\sigma_x^2}\right)f'^2 \\ &\quad +\varepsilon\left(1+\sigma_x^2\right)\theta' f''+\left(\frac{1-\sigma_x\tan\phi}{1+\sigma_x^2}\right)\theta \\ &= x\left(f'\frac{\partial f'}{\partial x}-f''\frac{\partial f}{\partial x}\right) \end{aligned} \tag{15}$$

$$\begin{aligned} &\frac{1}{Pr}\left(1+\sigma_x^2\right)(1+\gamma\theta)\theta''+\frac{1}{Pr}\left(1+\sigma_x^2\right)\gamma\theta'^2+\frac{7}{4}f\theta' \\ &= x\left(f'\frac{\partial \theta}{\partial x}-\theta'\frac{\partial f}{\partial x}\right) \end{aligned} \tag{16}$$

subject to the boundary conditions

$$\begin{aligned} &f(x,0)=f'(x,0)=0, \quad \theta(0,x)=1 \\ &f'(x,\infty)=\theta(0,\infty)=0 \end{aligned} \tag{17}$$

For flat cone ($a=0$) having constant viscosity and thermal conductivity equations (15) and (16) take the following form:

$$\begin{aligned} &(1+\varepsilon\theta)f'''+\frac{7}{4}ff''-\frac{1}{2}f'^2+\varepsilon\theta' f''+\theta \\ &= x\left(f'\frac{\partial f'}{\partial x}-f''\frac{\partial f}{\partial x}\right) \end{aligned} \tag{18}$$

$$\begin{aligned} &\frac{1}{Pr}(1+\gamma\theta)\theta''+\frac{1}{Pr}\gamma\theta'^2+\frac{7}{4}f\theta' \\ &= x\left(f'\frac{\partial \theta}{\partial x}-\theta'\frac{\partial f}{\partial x}\right) \end{aligned} \tag{19}$$

The boundary conditions remain the same.

3. RESULTS AND DISCUSSION

Equations (15) and (16) along with the boundary conditions (17) were solved numerically by the finite-difference method known as Keller box method. Since a good description of this method and its application to boundary-layer flow problems is given in the book by Cebeci and Bradshaw [24] as well as in many papers such as, for example, Hossain et al. [15, 16, 22, 23] it will not be presented here. Using this method we represent the results in terms of the skin friction coefficient c_f and the local Nusselt number Nu from the following relations:

$$c_f\left(\frac{Gr}{x}\right)^{1/4} = (1+\varepsilon)\left(1+\sigma_x^2\right)^{1/2} f''(x,0) \tag{20}$$

$$Nu\left(\frac{Gr}{x}\right)^{-1/4} = -(1+\gamma)\left(1+\sigma_x^2\right)^{1/2}\theta'(x,0) \tag{21}$$

We also calculate the average Nusselt number, $\overline{Nu}$, having the following relations:

$$\overline{Nu}\left(\frac{Gr}{x}\right)^{-1/4} = -(1+\gamma)\frac{1}{s}\int_0^x\left(1+\sigma_x^2\right)\theta'(x,0)\,\mathrm{d}x \tag{22}$$

where $s=\int_0^x(1+\sigma_x^2)\,\mathrm{d}x$.

Results are given for the variable viscosity parameter $\varepsilon=0.0$ (constant viscosity), 1.0, 2.0, 4.0 and 5.0; the variable thermal conductivity parameter $\gamma=0.0$ (constant thermal conductivity), 1.0, 2.0, 4.0 and 5.0; the amplitude parameter $a=0.0$ (flat cone) and 0.3; Prandtl number $Pr=0.7$ (air); cone half-angle $\phi=0°$ (flat plate), 30° and 45°.

In *tables I* and *II* we entered the values of $f''(\xi,0)$ and $-\theta'(\xi,0)$, respectively, showing the variation of Pr, ε and γ. From which we can conclude that if the value of Pr increases the values of $f''(\xi,0)$ decreases and

TABLE I
The values of $f''(\xi, 0)$ for different Pr with $a = 0.3$ and $\phi = 30°$.

Pr	$\varepsilon = 0.0$		$\varepsilon = 5.0$	
	$\gamma = 0.0$	$\gamma = 5.0$	$\gamma = 0.0$	$\gamma = 5.0$
0.1	0.06391	0.07332	0.01303	0.01488
0.7	0.06252	0.07286	0.01294	0.01487
1.0	0.06187	0.07270	0.01290	0.01486
7.0	0.05287	0.06857	0.01212	0.01467

TABLE II
The values of $-\theta'(\xi, 0)$ for different Pr with $a = 0.3$ and $\phi = 30°$.

Pr	$\varepsilon = 0.0$		$\varepsilon = 5.0$	
	$\gamma = 0.0$	$\gamma = 5.0$	$\gamma = 0.0$	$\gamma = 5.0$
0.1	0.16848	0.09755	0.16717	0.09730
0.7	0.17882	0.09988	0.17005	0.09783
1.0	0.18375	0.10063	0.17147	0.09809
7.0	0.25734	0.12072	0.19752	0.10344

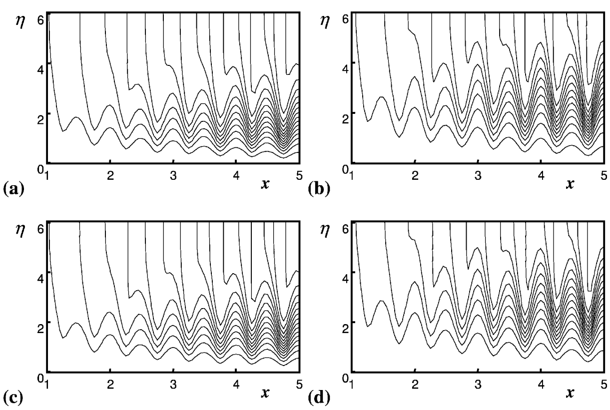

Figure 2. Dimensionless streamlines for $Pr = 0.7$, $\phi = 30°$ and $a = 0.3$ while (a) $\varepsilon = 0.0$, $\gamma = 0.0$, (b) $\varepsilon = 5.0$, $\gamma = 0.0$, (c) $\varepsilon = 0.0$, $\gamma = 5.0$ and (d) $\varepsilon = 5.0$, $\gamma = 5.0$.

$-\theta'(\xi, 0)$ increases for any value of ε and γ. We also see that if the value of ε increases the value of $f''(\xi, 0)$ decreases while it increases for increase of γ and the value of $-\theta'(\xi, 0)$ decreases for increase in both ε and γ.

Figures 2 and *3* illustrate the effects of the parameters ε and γ on the development of streamlines and isotherms which are plotted at identical intervals for a wavy cone ($a = 0.3$). When $\varepsilon = \gamma = 0.0$ we recover the problem discussed by Pop and Na [9], where the fluid viscosity and thermal conductivity are independent of temperature. In *figure 2* we see that an increase in the value of ε

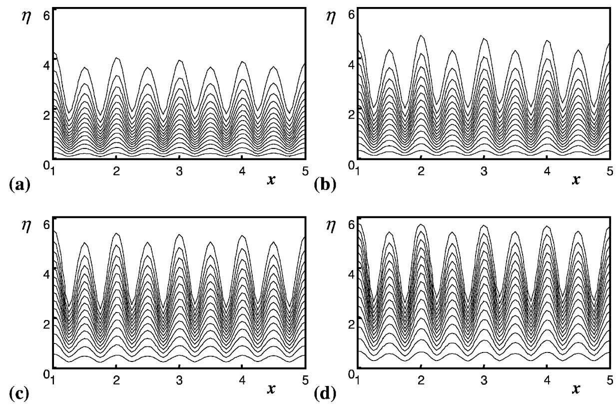

Figure 3. Dimensionless isotherms for $Pr = 0.7$, $\phi = 30°$ and $a = 0.3$ while (a) $\varepsilon = 0.0$, $\gamma = 0.0$, (b) $\varepsilon = 5.0$, $\gamma = 0.0$, (c) $\varepsilon = 0.0$, $\gamma = 5.0$ and (d) $\varepsilon = 5.0$, $\gamma = 5.0$.

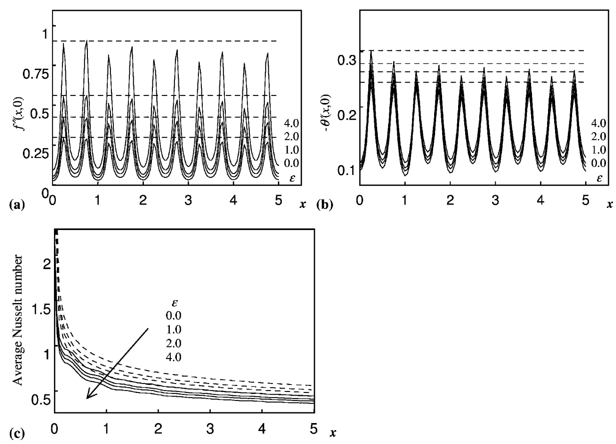

Figure 4. Variation of of (a) the reduced skin friction $f''(x, 0)$, (b) reduced heat transfer $-\theta'(x, 0)$ and (c) average Nusselt number with x for different values of ε with $Pr = 0.7$, $\gamma = 1.0$ and $\phi = 30°$.

causes the effects of the wavy surface to be attenuated and the boundary layer becomes thinner. On the other hand, similar thing happens when the value of γ decreases. From *figure 3* we see that the boundary layer maintains its overall thickness in terms of η when x is large.

Variation of the reduced skin friction coefficient $f''(x, 0)$, reduced heat transfer $-\theta'(x, 0)$ and the average Nusselt number $\overline{Nu}$ as a function of x is shown in *figures 4–6* for fluids with $Pr = 0.7$, $a = 0.3$ and selected values of the parameters ε, γ and ϕ. It is seen that, as expected, the values of $f''(x, 0)$, $-\theta'(x, 0)$ and $\overline{Nu}$ are lower for a wavy cone than for a flat cone. When the heated surface of the cone is not flat ($a \neq 0$), the component of the buoyancy force along the cone is reduced by a factor $(1 - \sigma_x \tan\phi)/(1 + \sigma_x^2)$, as shown in equa-

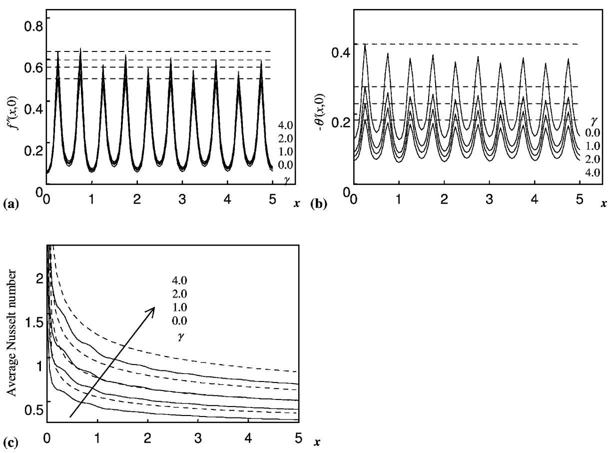

Figure 5. Variation of (a) the reduced skin friction $f''(x,0)$, (b) reduced heat transfer $-\theta'(x,0)$ and (c) average Nusselt number with x for different values of γ with $Pr = 0.7$, $\varepsilon = 1.0$ and $\phi = 30°$.

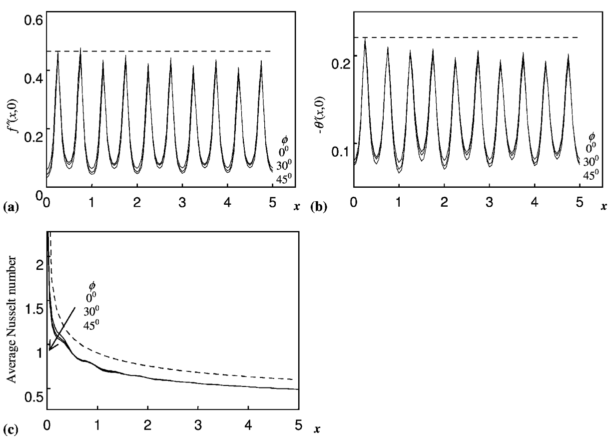

Figure 6. Variation of (a) the reduced skin friction $f''(x,0)$, (b) reduced heat transfer $-\theta'(x,0)$ and (c) average Nusselt number with x for different values of ϕ with $Pr = 0.7$, $\varepsilon = 2.0$ and $\gamma = 2$.

tion (15), from its maximum value of a flat cone. Consequently, the boundary layer thickness is locally smaller, and hence, local rates of skin friction coefficient and heat transfer are reduced. The changes are seen to be more pronounced for larger cone angles, as seen from *figure 6*. Further, *figure 4* shows that an increase in the variable viscosity parameter ε leads to a decrease of the reduced skin friction heat transfer rates and the value of average Nusselt number from the cone. Again, *figure 5* shows that an increase in the variable thermal conductivity parameter γ leads to an increase of the reduced skin friction and to increase of the heat transfer rates and the average Nusselt number from the cone.

4. CONCLUSIONS

A theoretical study of the laminar free convection boundary-layer heat transfer between a vertical wavy cone with a constant surface temperature and a fluid of variable viscosity and thermal conductivity has been made in this paper. New variables to transform the complex geometry into a simplex shape were used and a very efficient implicit finite-difference (Keller box) scheme was employed to solve the boundary-layer equations. It has been found that the effect of increasing the viscosity parameter ε results in decreasing the skin friction coefficient, heat transfer rate and the average Nusselt number. When the effect of thermal conductivity parameter is included, the heat transfer rate and the average Nusselt number play the same role but the skin friction coefficient increases with the increase of thermal conductivity. It is worth mentioning that the amplitude of the waves must be within an $O(Gr^{-1/4})$ range in order to balance the direct and indirect buoyancy forces. Strong enough curvature may produce flow separation, or rather, the flow develops a region of reverse flow at the surface of the wavy cone. Rees and Pop [6] recently presented a detailed study of this flow behavior.

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